On the existence and properties of solitary waves traveling in tensegrity-like lattices

Group Meeting Prof. Julian Rimoli,
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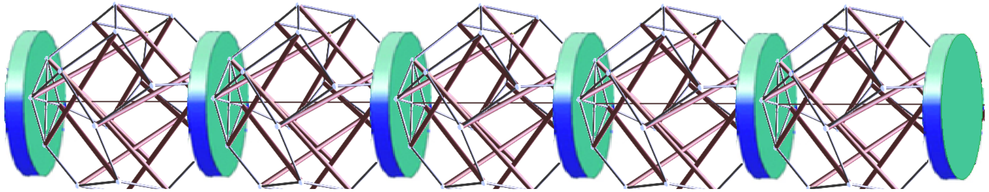

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Outline

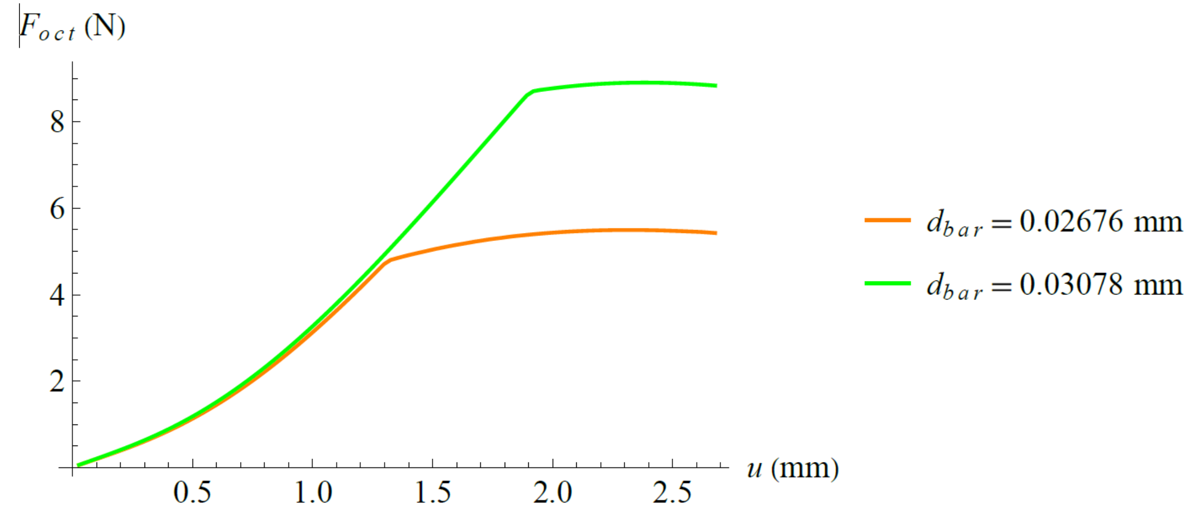
- Introduction
- Mechanical response of an octahedron tensegrity unit
- A tensegrity-like potential for the truncated octahedron
- Wave propagation problem
- Numerical simulations
- Conclusions

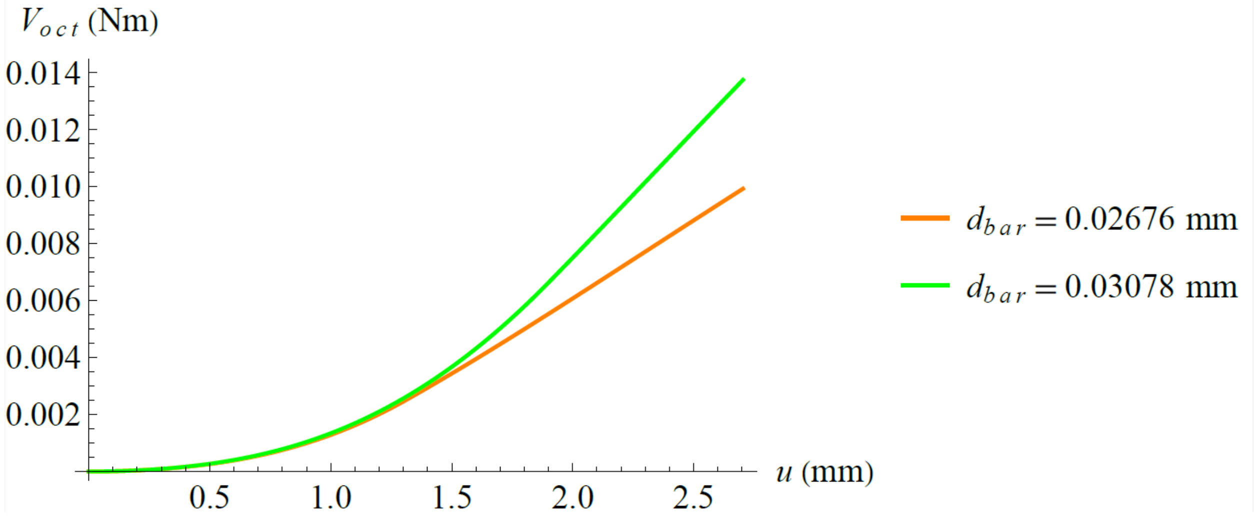
Introduction

- Based on previous work [1], we wish to prove the existence of solitary waves in a 1-D metamaterial formed by truncated octahedrons;
- The analytical approach consists firstly of finding a non-linear spring with behavior on compression that is similar to the one of a truncated octahedron;
- With the tensegrity-like law, the equations of motion for the quasi-continuum limit for a mass-spring chain are derived and a Weierstrass analysis is used to prove the existence of compression solitary waves;
- Numerical simulations on a tensegrity-like mass-spring chain are used to validate the analytical approach

Mechanical response of an octahedron tensegrity unit

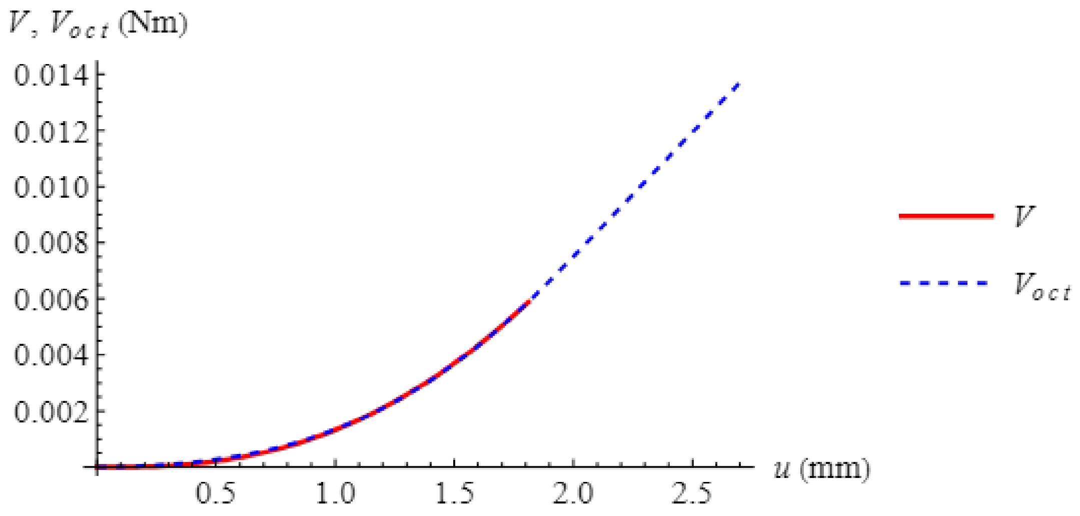
- Stiffness increases until a maximum value and F(u) tends to be linear. This behavior is valid until the point where the bars buckle





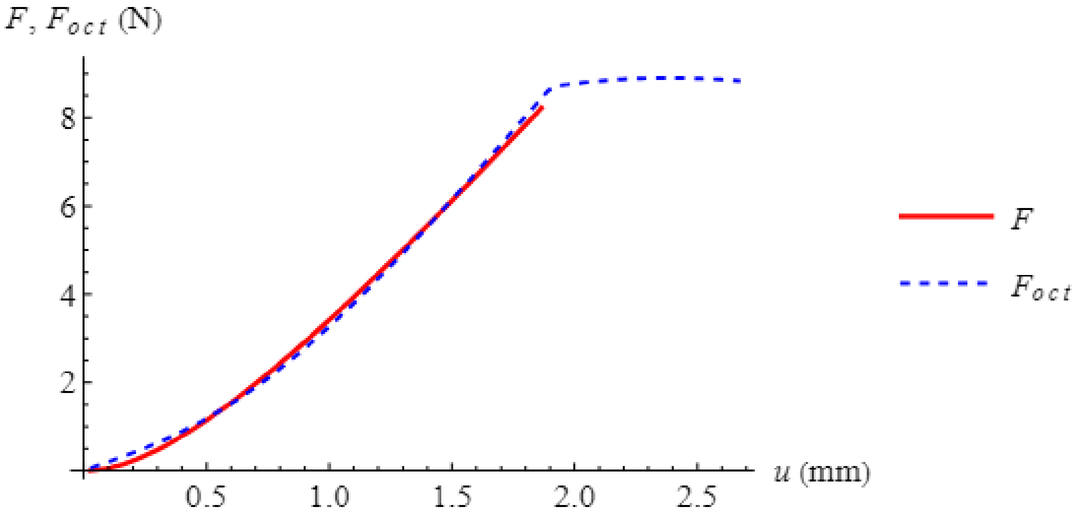
A tensegrity-like potential for the truncated octahedron

$$V(u) = \alpha_1 \, u^2 \, \frac{u + \alpha_2}{u + \alpha_3}$$



(a)

$$F(u) = \frac{dV}{du}$$



Wave propagation problem

Quasi-continuum limit of the equations of motion

- Equation of motion of the *i*-th mass

$$m\ddot{u}_i = V_u(u_{i-1} - u_i) - V_u(u_i - u_{i+1})$$

- Quasi-continuum approach -> Boussinesq equation:

$$m u_{tt} = [V_u]_{xx} + \gamma u_{xxxx}$$

$$\xi_{tt} = \left[\tilde{V}_\xi\right]_{xx} + \gamma \xi_{xxxx} \qquad \tilde{V}(\xi) = V(\xi)/(m u_{lock}^2)$$

- Traveling wave $\xi = \Phi(x - vt) = \Phi(z)$ (v -> wave speed)

$$v^2 \Phi'' = [\tilde{V}_\Phi]'' + \gamma \Phi''''$$

- Integrating twice with respect to time

$$v^2 \Phi = \tilde{V}_\Phi + \gamma \Phi''$$

Wave propagation problem

Quasi-continuum limit of the equations of motion

- Multiplying both sides by Φ', integrating with respect to z and setting $\mathcal{F} = \gamma \Phi'^2$, one obtains

$$\mathscr{F} = v^2 \phi^2 - 2\tilde{V} + C$$

  that is, making use of the expression of $\tilde{V}$

$$\mathscr{F} = \phi^2 \left[v^2 - 2\, \tilde{\alpha}_1 \frac{\tilde{\alpha}_2 + \phi}{\tilde{\alpha}_3 + \phi} \right]$$

- C: integration constant
- $\tilde{\alpha}_1 = \frac{\alpha_1 h_0^2}{m}, \tilde{\alpha}_2 = \frac{\alpha_2}{h_0}, \tilde{\alpha}_3 = \frac{\alpha_3}{h_0}$
- The existence of solitary waves can be proven by a Weisstress discussion:
  - Φ needs to admit two asymptotic zero points and an intermediate inversion point
  - We shall find the zeros of Φ' -> zeros of F

Wave propagation problem

Existence of the solitary pulse

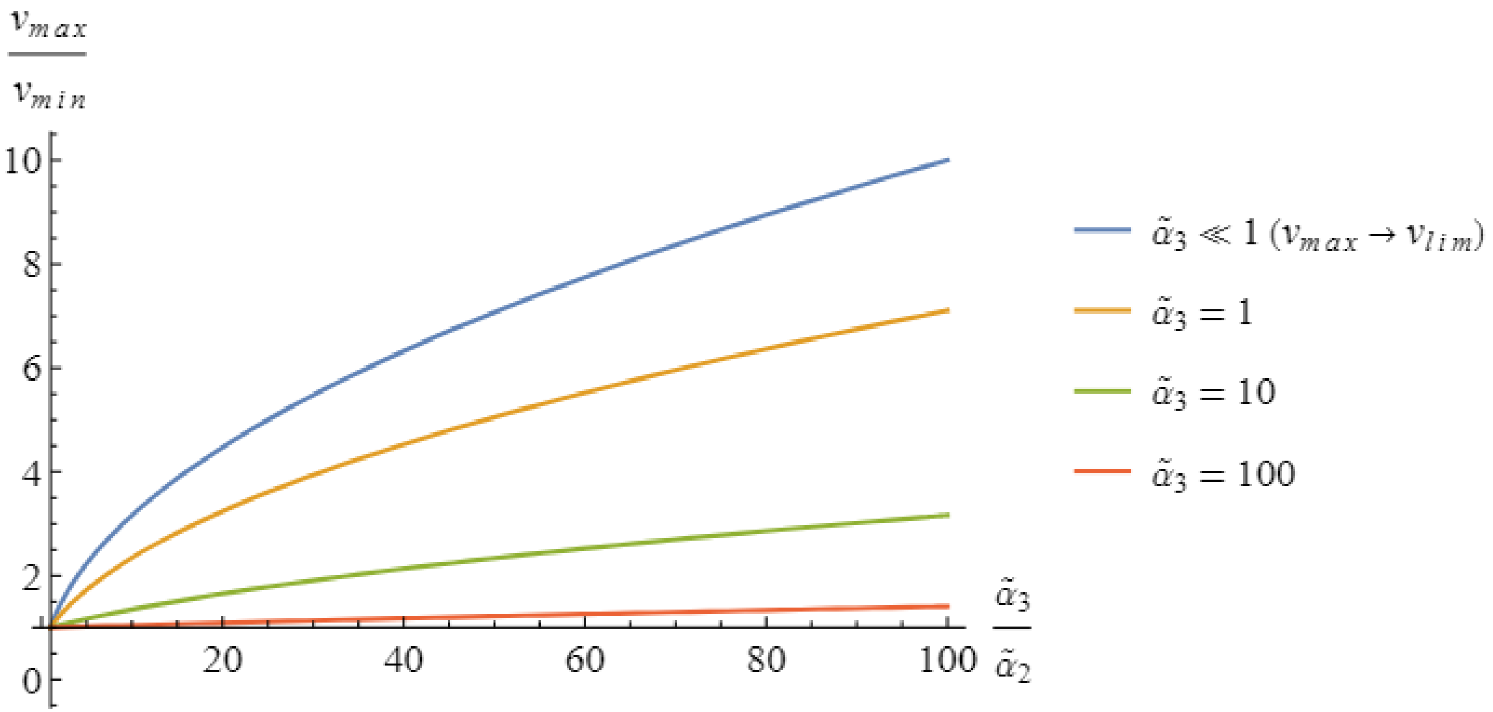
- Analysing the zeros of $\mathcal{F}$

$$\mathscr{F} = \phi^2 \left[v^2 - 2\ \tilde{\alpha}_1 \frac{\tilde{\alpha}_2 + \phi}{\tilde{\alpha}_3 + \phi} \right]$$

- $\Phi = 0$ -> two asymptotic zero points
- $\phi^* = \dfrac{\tilde{\alpha}_3 v^2 - 2\tilde{\alpha}_1\tilde{\alpha}_2}{2\tilde{\alpha}_1 - v^2}$ $\quad\quad \boldsymbol{\Phi}^* \in]0, 1]$
- $\Phi^* = 1$

Wave speed range

$$v_{min} = \sqrt{2\tilde{\alpha}_1 \frac{\tilde{\alpha}_2}{\tilde{\alpha}_3}} = h_0 \sqrt{\frac{k_0}{m}} = c_0 \qquad v_{max} = \sqrt{2\tilde{\alpha}_1 \frac{1+\tilde{\alpha}_2}{1+\tilde{\alpha}_3}} \qquad v_{lim} = \sqrt{2\tilde{\alpha}_1}$$



Wave profiles

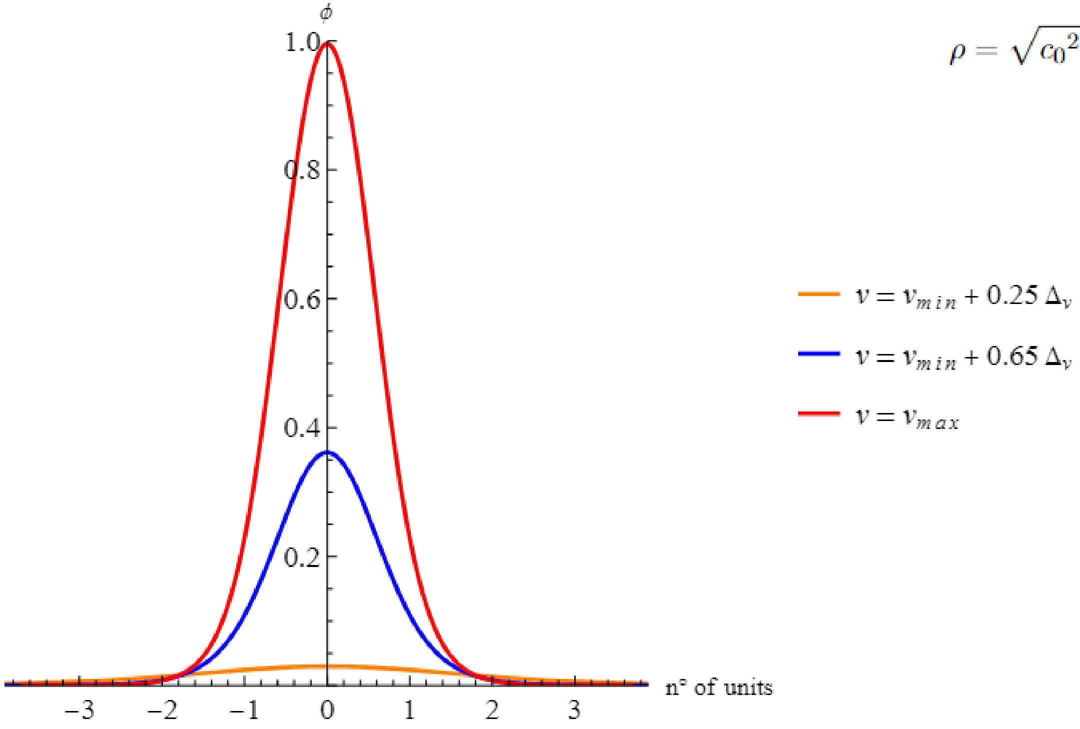
$$\frac{d\phi}{dz} = \sqrt{\frac{\mathscr{F}}{\gamma}}$$

$$f := \int \sqrt{\frac{\gamma}{\mathscr{F}}}\, d\phi = \pm z$$

$$\boxed{f = c_0 h_0 \frac{\sqrt{3}}{3} \frac{\psi \tan^{-1}\left(\frac{\chi}{\rho}\right) - \rho \tan^{-1}\left(\frac{\chi}{\psi}\right)}{\psi \rho}}$$

$$\chi = \sqrt{v^2 - 2{h_0}^2 \alpha_1 \frac{\alpha_2 + u_{buck}\ \phi}{m\ (\alpha_3 + u_{buck}\ \phi)}},$$

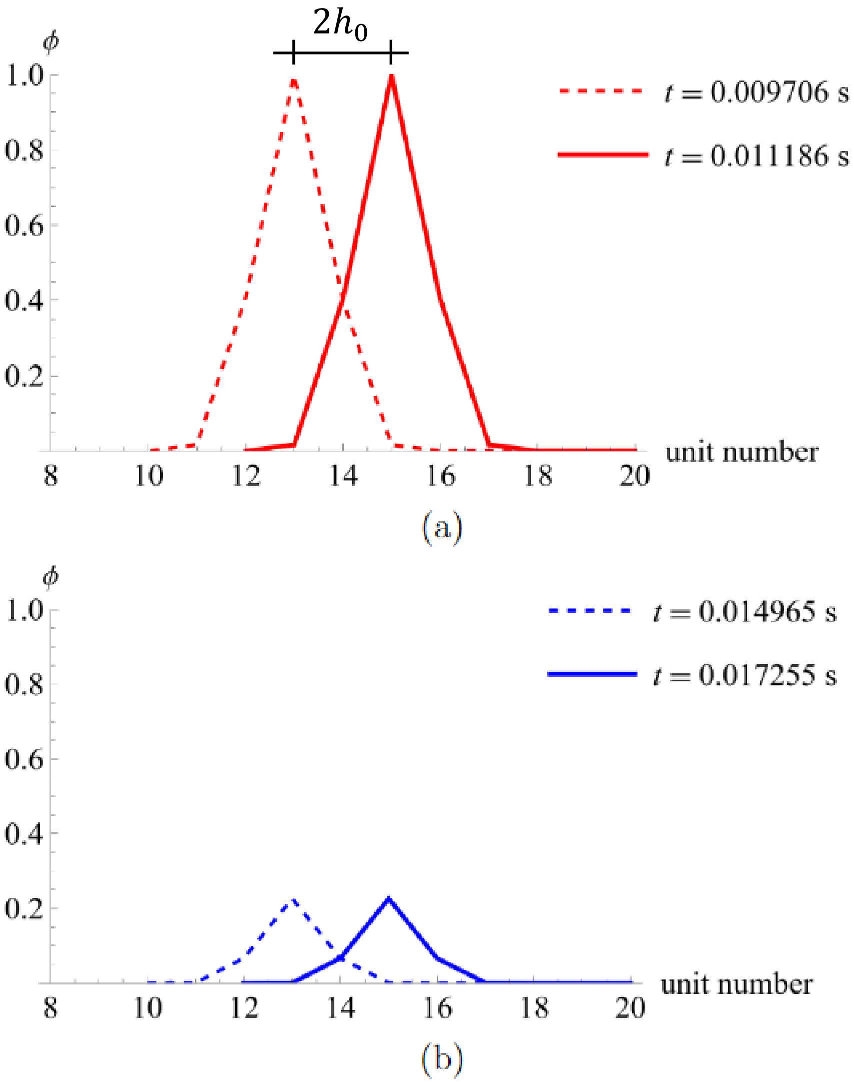
$$\rho = \sqrt{{c_0}^2 - v^2}, \qquad \psi = \sqrt{{v_{lim}}^2 - v^2}$$



Numerical simulations

Numerical wave speed:

$$v = 2h_0/\Delta t$$



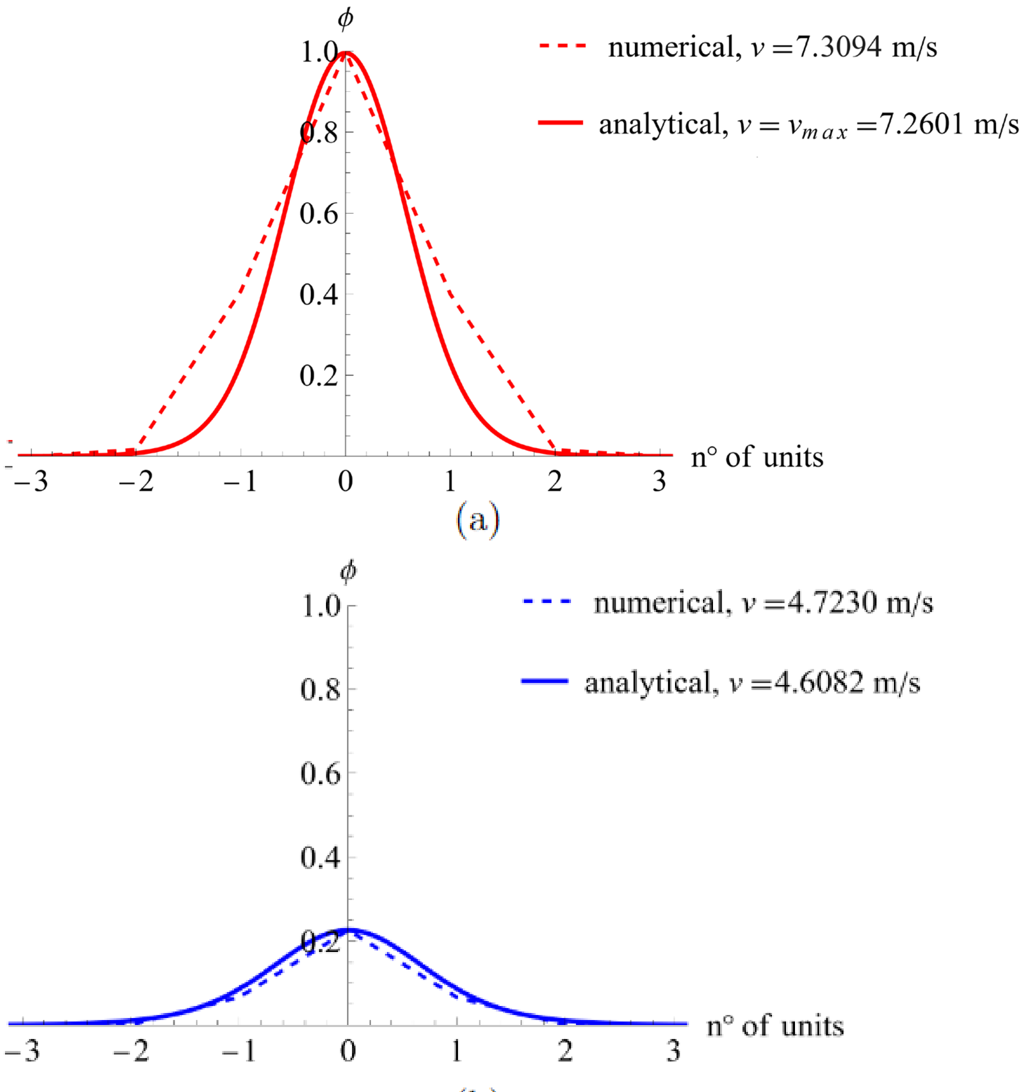
(a)

(b)

Numerical simulations

Amplitude vs. wave speed:

$$\phi^* = \frac{\tilde{\alpha}_3 v^2 - 2\tilde{\alpha}_1\tilde{\alpha}_2}{2\tilde{\alpha}_1 - v^2}$$



(a)

(b)

Conclusions

- An analytic characterization of the solitary pulses that propagate in truncated octahedrons tensegrity-like lattices was provided.

- The shape of the pulses traveling through such systems has been derived in implicit form using the Weierstrass theory of one-dimensional Lagrangian systems.

- The stiffness of the non-linear spring presents an asymptotic behavior. Interestingly, the maximum wave speed also tends to have a limit value.

- The relation between wave speed and amplitude was confirmed by numerical simulations.

- The localization behavior for increasing values of speed observed in [1] was not observed for these tensegrity-like springs.

[1] A. Amendola. "An analytic study on the properties of solitary waves traveling on tensegrity-like lattices". In: International Journal of Non-Linear Mechanics 148 (2023), 104264

Thank you for your attention!