

On the use of fractal geometry for the design of tensegrity braces

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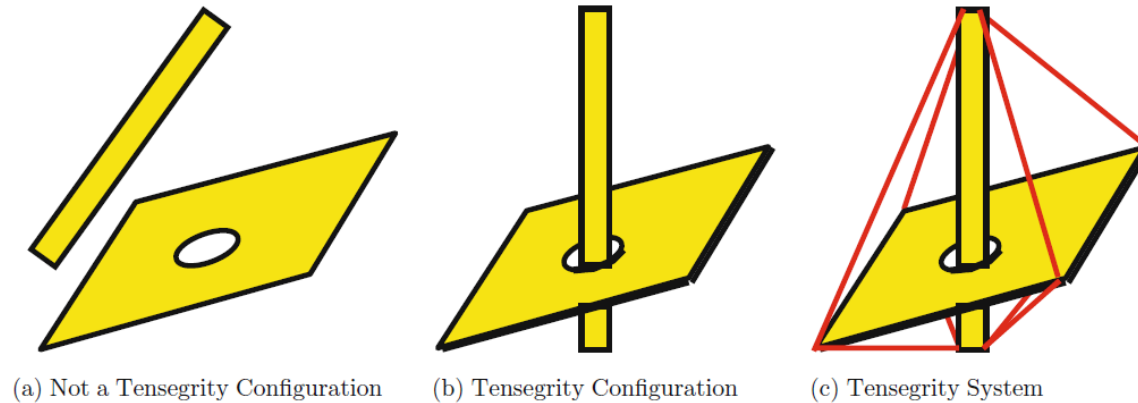
This work was supported by the Italian Ministry of Foreign Affairs and International Cooperation", grant number US23GR15

Outline



- Introduction and Motivation
- SMAD Braces
- Mass reduction and displacement amplification
- Constitutive behavior of the SMA wires
- Use of SMAD braces in anti-seismic structures
- Use in timber frames
- Form-finding procedure based on fractal geometry
- SMAD metamaterials
- Concluding remarks

Introduction and Motivation



- In the absence of external forces, let a set of rigid bodies in a specific configuration have torque less connections (e.g. via frictionless ball-joints).
- Then this configuration forms a tensegrity configuration if the system of rigid bodies can be stabilized by some set of internal tensile members, i.e. connected between the rigid bodies.
- The configuration is not a tensegrity configuration if no tensile members are required and/or no set of tensile members exist to stabilize the configuration.

(Skelton and de Oliveira, Tensegrity Systems, Springer 2010)

Tensegrity ubiquity

..in Nature



(a) class 2 shoulder joint and class 3 elbow joint



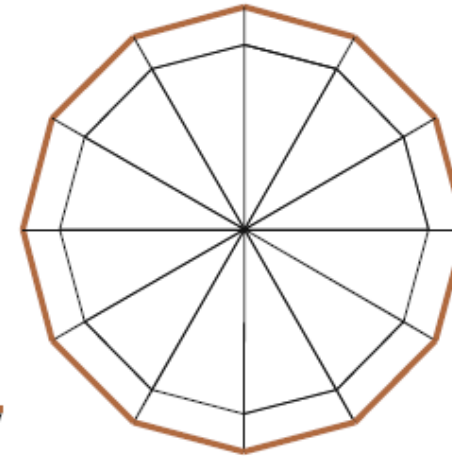
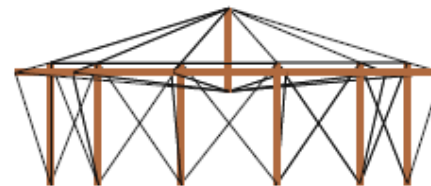
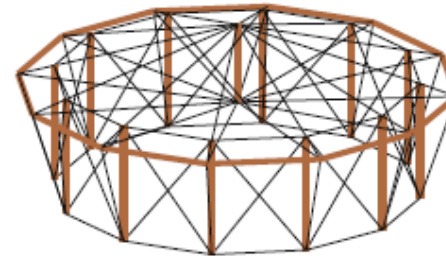
(b) class 2 toe joints

..in Art



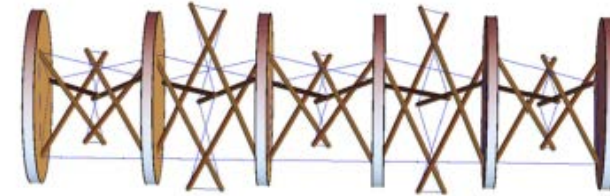
“Needle Tower 2”, Kenneth Snelson, Holland

..in Engineering and Architecture

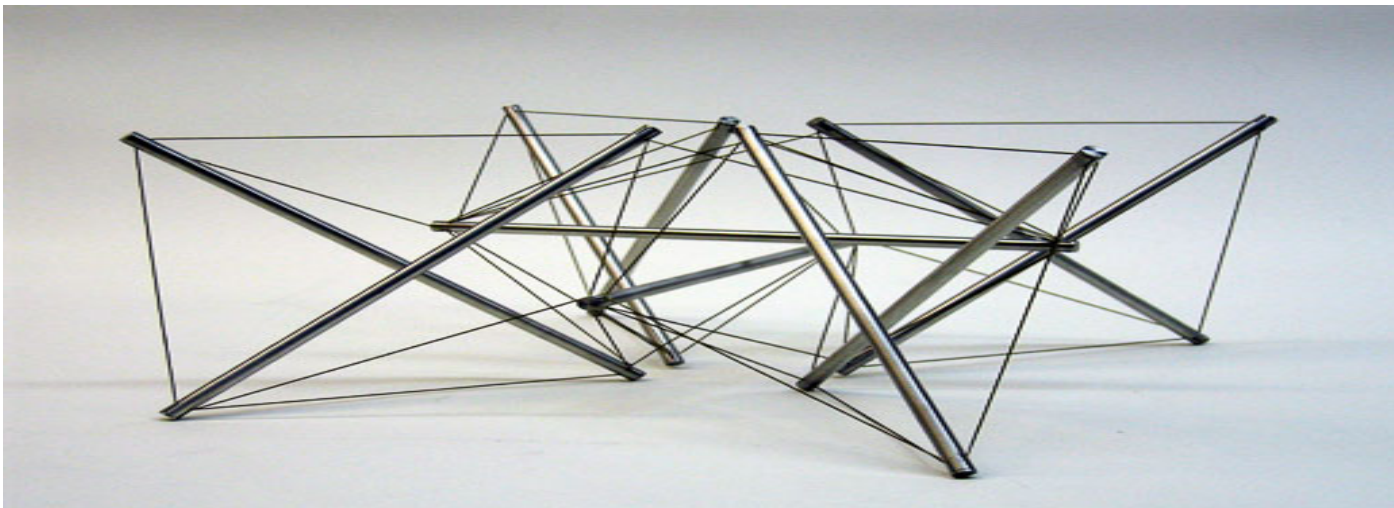


Nonlinearity and Deployability

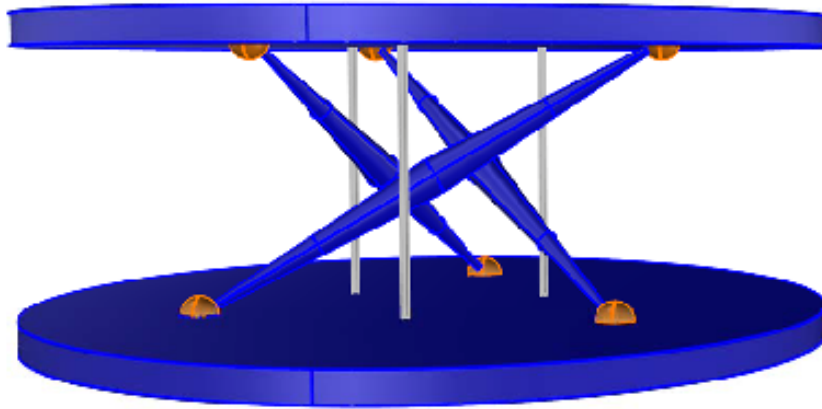
A novel area of research has emerged over the last few years regarding the highly nonlinear dynamics of mechanical metamaterials formed by assembling tensegrity units and lumped masses. Experimental and theoretical studies have shown that the geometrically nonlinear response of several tensegrity units may gradually change from stiffening to softening, depending on mechanical, geometrical and prestress variables. Such a behavior, which can be dynamically tuned, supports highly-controllable bandgap and solitary wave dynamics.



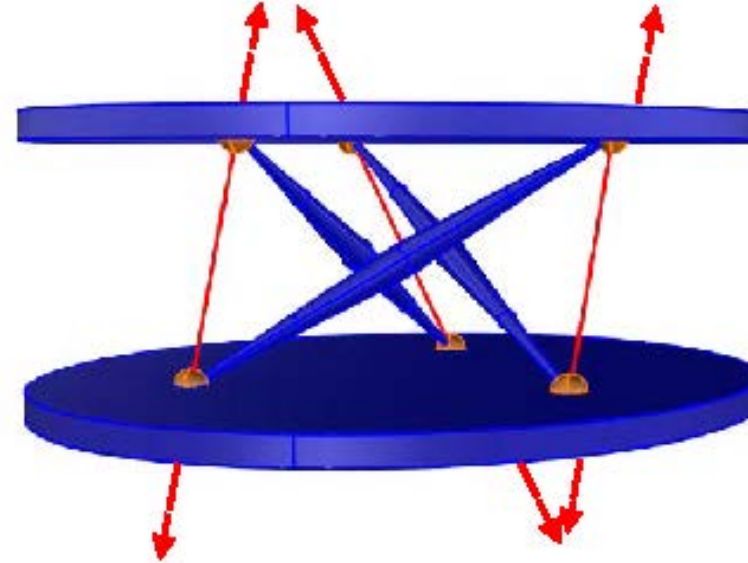
Deployable tensegrity structure
(courtesy of Bob Skelton)



3D printing of tensegrity structures



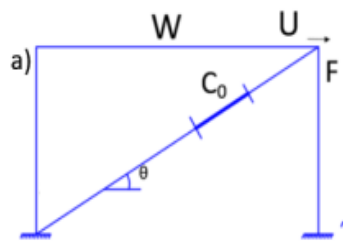
Graphical model of a building block equipped with provisional supports



Building block at the end of the post-tensioning phase

Amendola, A., Nava, E.H., Goodall, R., Todd, I., Skelton, R.E., Fraternali, F. On the additive manufacturing, post-tensioning and testing of bi-material tensegrity structures. COMPOSITE STRUCTURES, 131, 66-71, 2015.

Conventional dissipative bracing systems

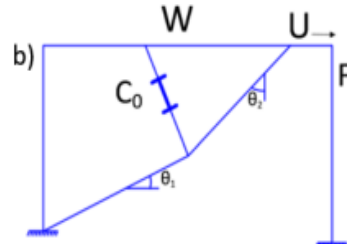


$$\theta = 37^\circ$$

$$f = \cos\theta$$

$$\beta = 0.03$$

$$f = \cos\theta$$



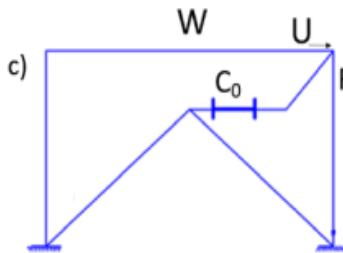
$$f = \frac{\sin\theta_1}{\cos(\theta_1 + \theta_2)} + \sin\theta_1$$

$$\theta_1 = 31.9^\circ$$

$$\theta_2 = 43.2^\circ$$

$$f = 3.191$$

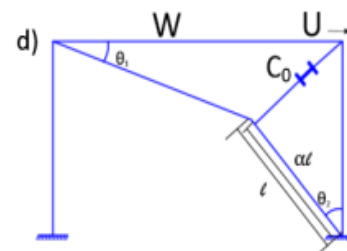
$$\beta = 0.509$$



$$\theta = 37^\circ$$

$$f = 1$$

$$\beta = 0.05$$



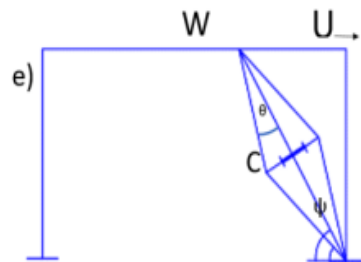
$$f = \frac{\alpha \cos\theta_1}{\cos(\theta_1 + \theta_2)} - \cos\theta_2$$

$$\theta_1 = 30^\circ$$

$$\theta_2 = 49^\circ$$

$$\alpha = 0.7$$

$$\beta = 0.318$$



$$f = \frac{\cos\psi}{\tan\theta}$$

$$\theta = 9^\circ$$

$$\beta = 0.23$$

$$\psi = 70^\circ$$

$$f = 2.16$$

- Scissor-jack configuration (e) can achieve magnification factors f (ratio between the displacement exhibited by the dissipative element and the lateral displacements of the frame) that are substantially greater than unity.
- Toggle-brace-damper (C & d) systems can also achieve the magnification factor greater than the unity.

Tensegrity Braces for Energy Dissipation

SMAD braces

D&M of a tensegrity structure equipped with SMA cables as an anti-seismic bracing system ('SMAD brace').

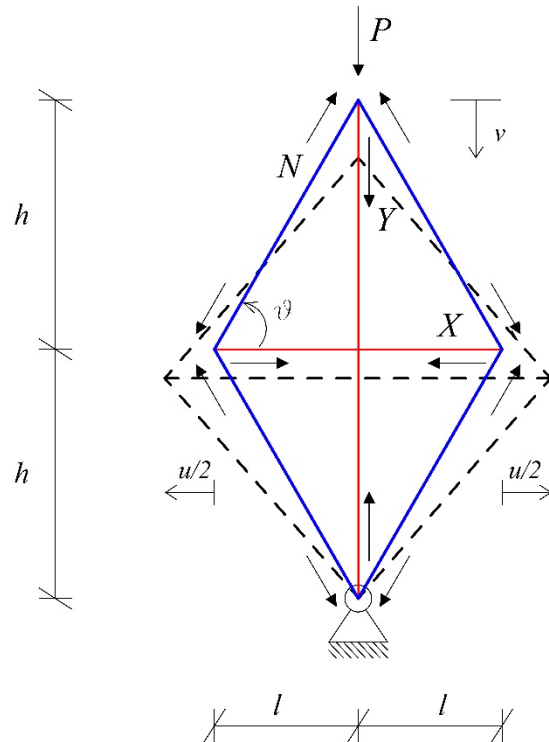
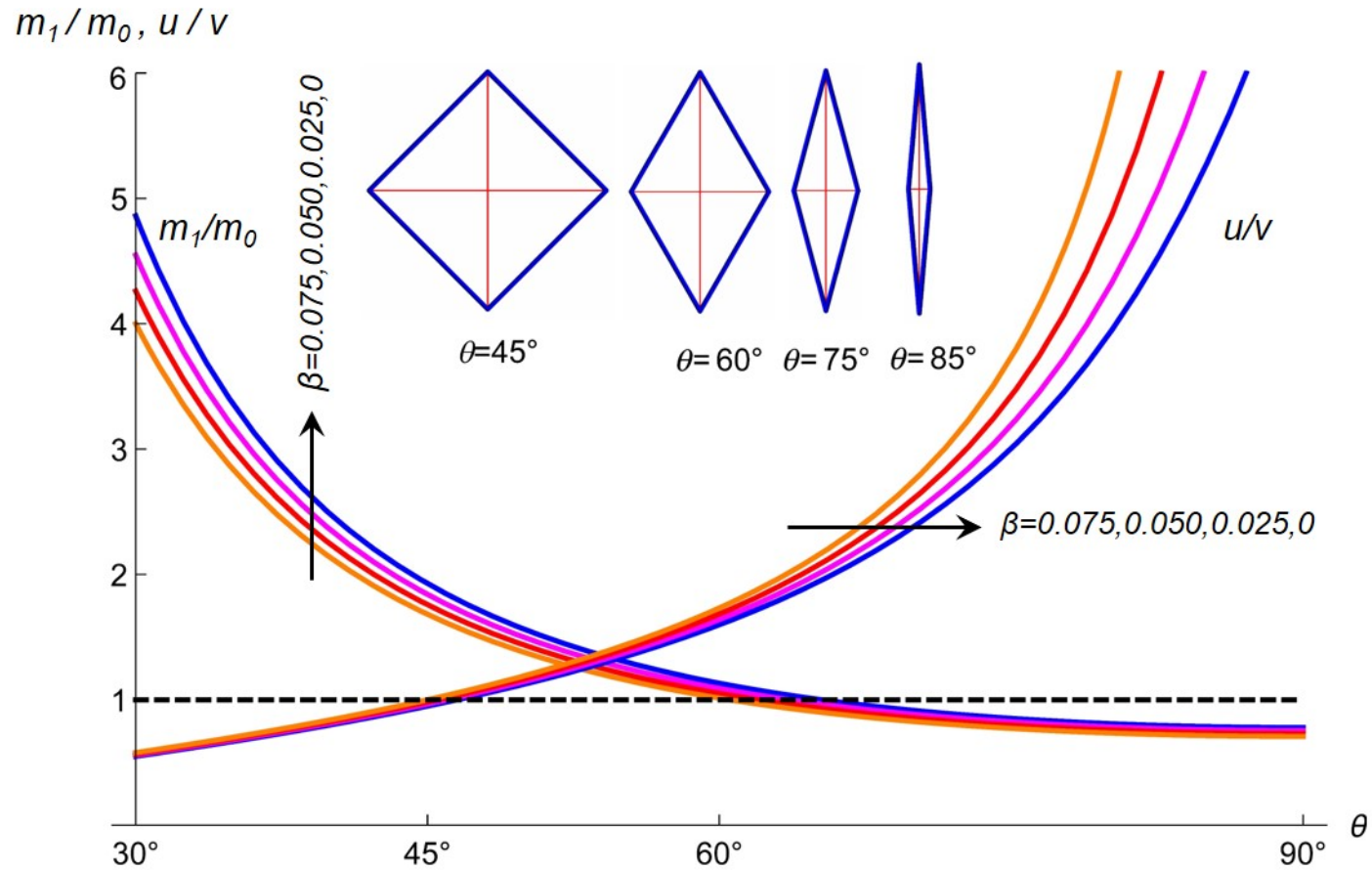


Illustration of the reference (solid lines) and deformed (dashed lines) configurations of the SMAD brace

Reference: **Fraternali, F., Santos, F.** Mechanical modeling of superelastic tensegrity braces for earthquake-proof structures. *EXTREME MECHANICS LETTERS*, 33, 100578, 2019. ISSN:[2352-4316](https://doi.org/10.1016/j.eml.2019.100578), DOI: <https://doi.org/10.1016/j.eml.2019.100578> - Open access preprint: [arXiv:1910.07080](https://arxiv.org/abs/1910.07080) [physics.app-ph]

Mass reduction and displacement amplification



$$m_1 = 4 \times 2\ell_1^2 \sqrt{N_{cr}/(\pi E)} = m_0 (2 \sin^5 \hat{\theta})^{-\frac{1}{2}}$$

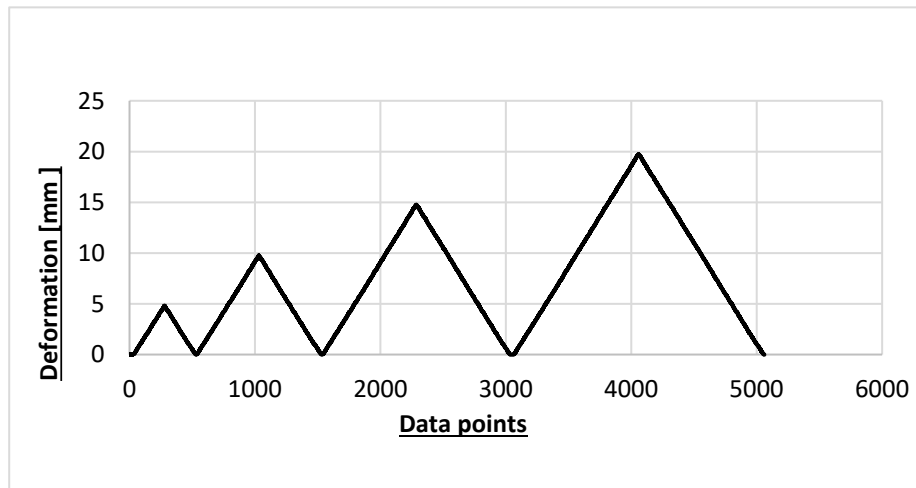
$$u/v = (\sqrt{4 \cos^2(\theta) - \beta(\beta - 4 \sin(\theta))} - 2 \cos(\theta)) / \beta$$

Constitutive behavior of the SMA wires

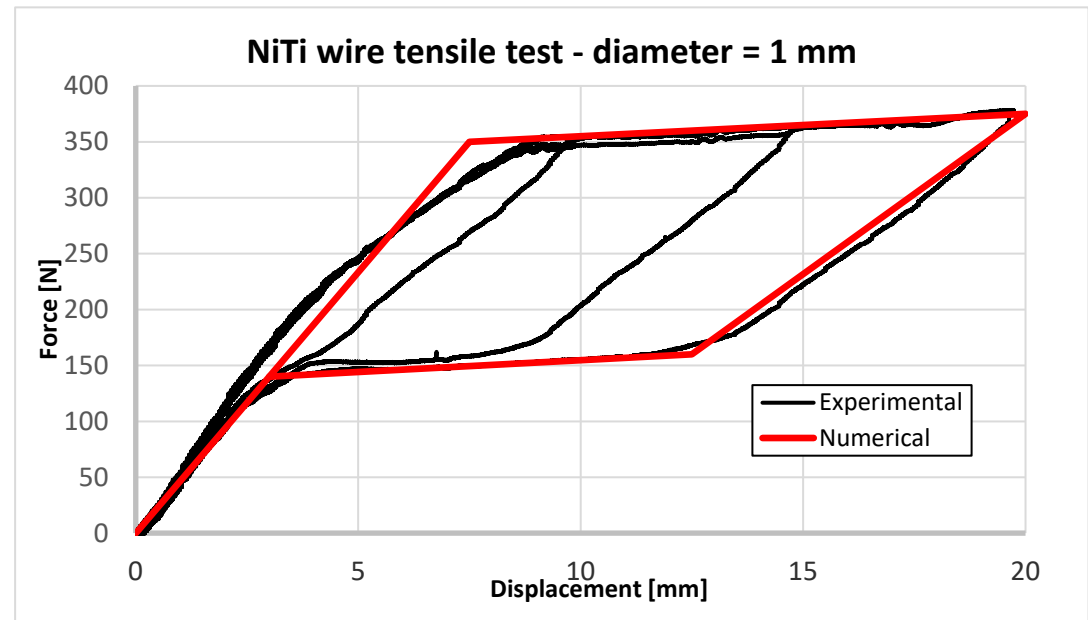
NITINOL AS A SUPERELASTIC MATERIAL

- Nitinol is a nickel-titanium (Ni-Ti) alloy known for its unique properties of superelasticity and shape memory effect.
- Specimen tested had an Activation temperature (Austenite finish temperature) $A_f \pm 10^\circ \text{C}$
- Effective Length of specimen tested- **725mm** Diameter- **1mm**

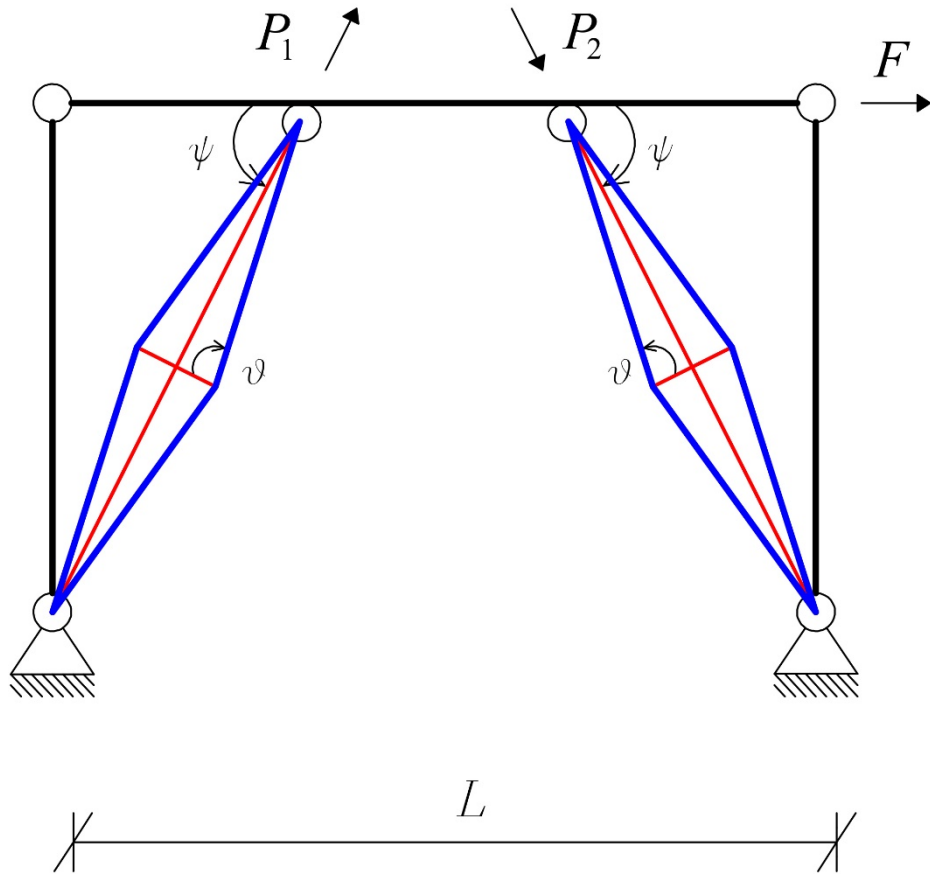
Constant Loading rate 0.1mm/sec



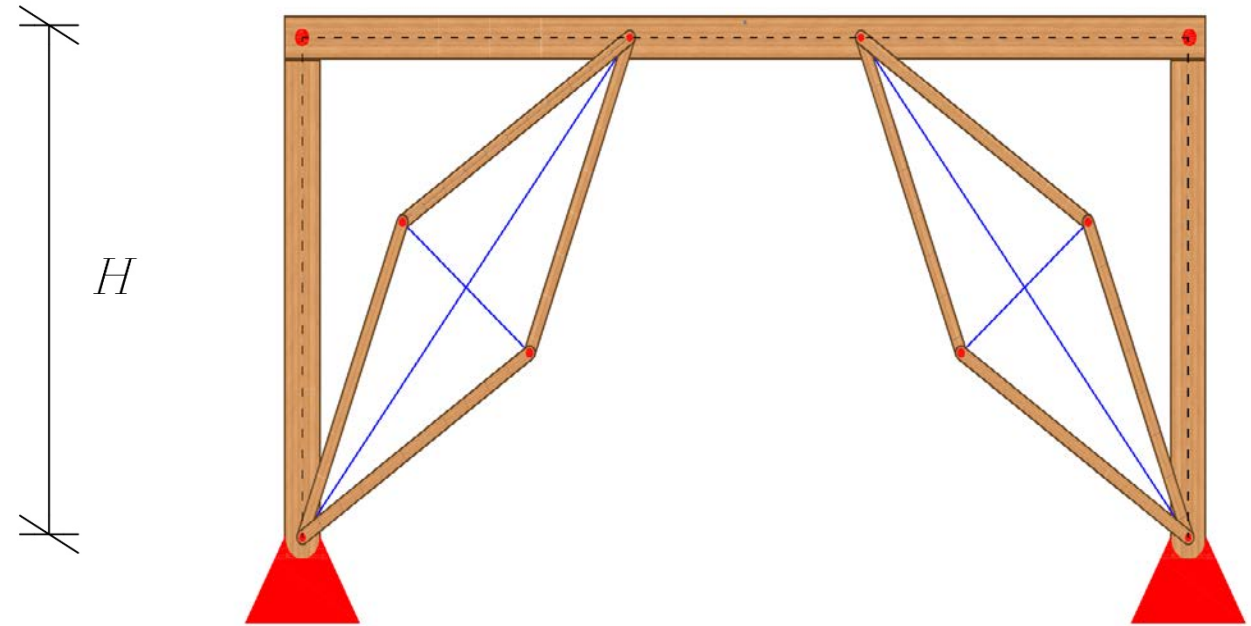
Tensile Test of Nitinol Wire



Use of SMAD braces in anti-seismic structures

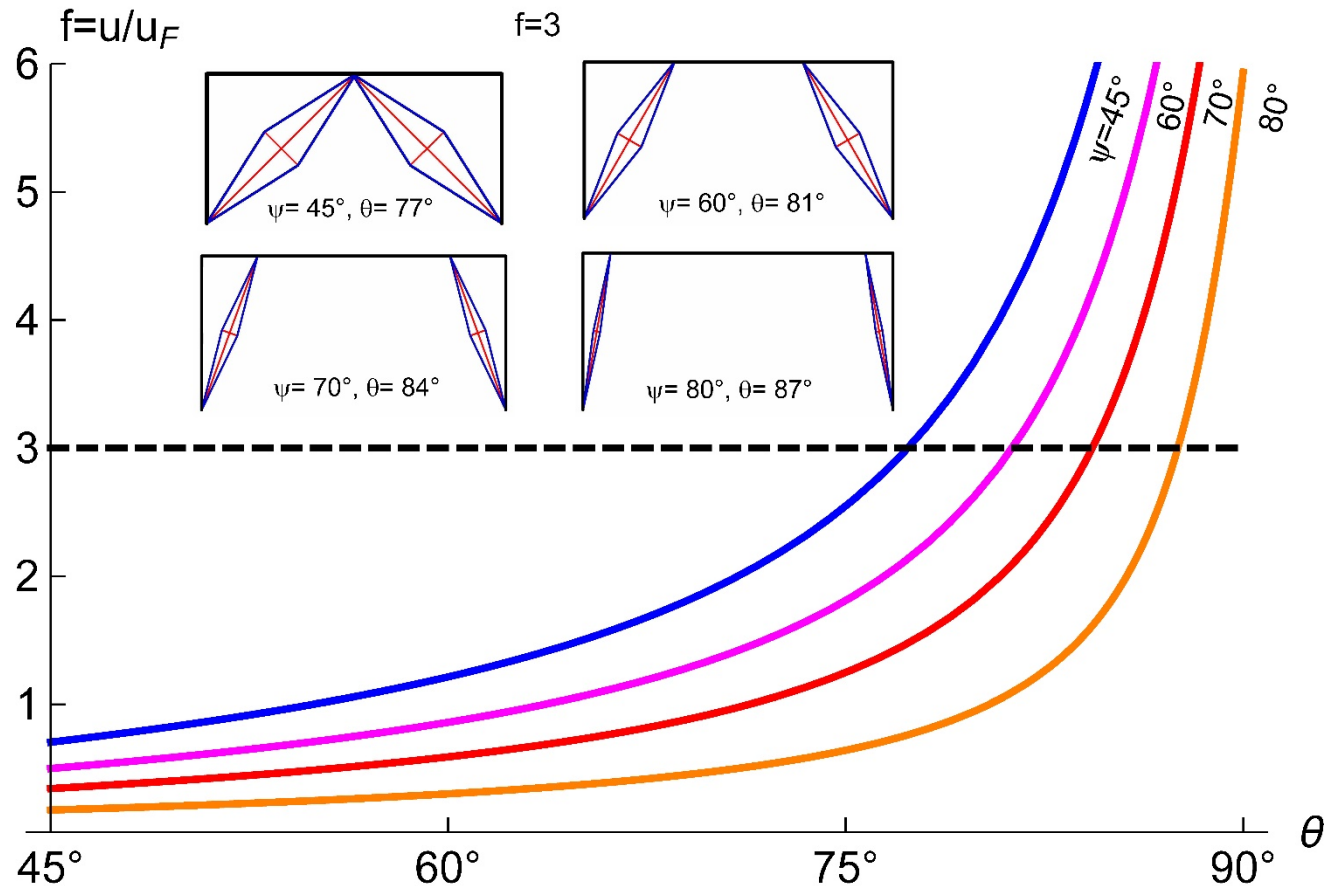


Frame structure strengthened with symmetric SMAD braces



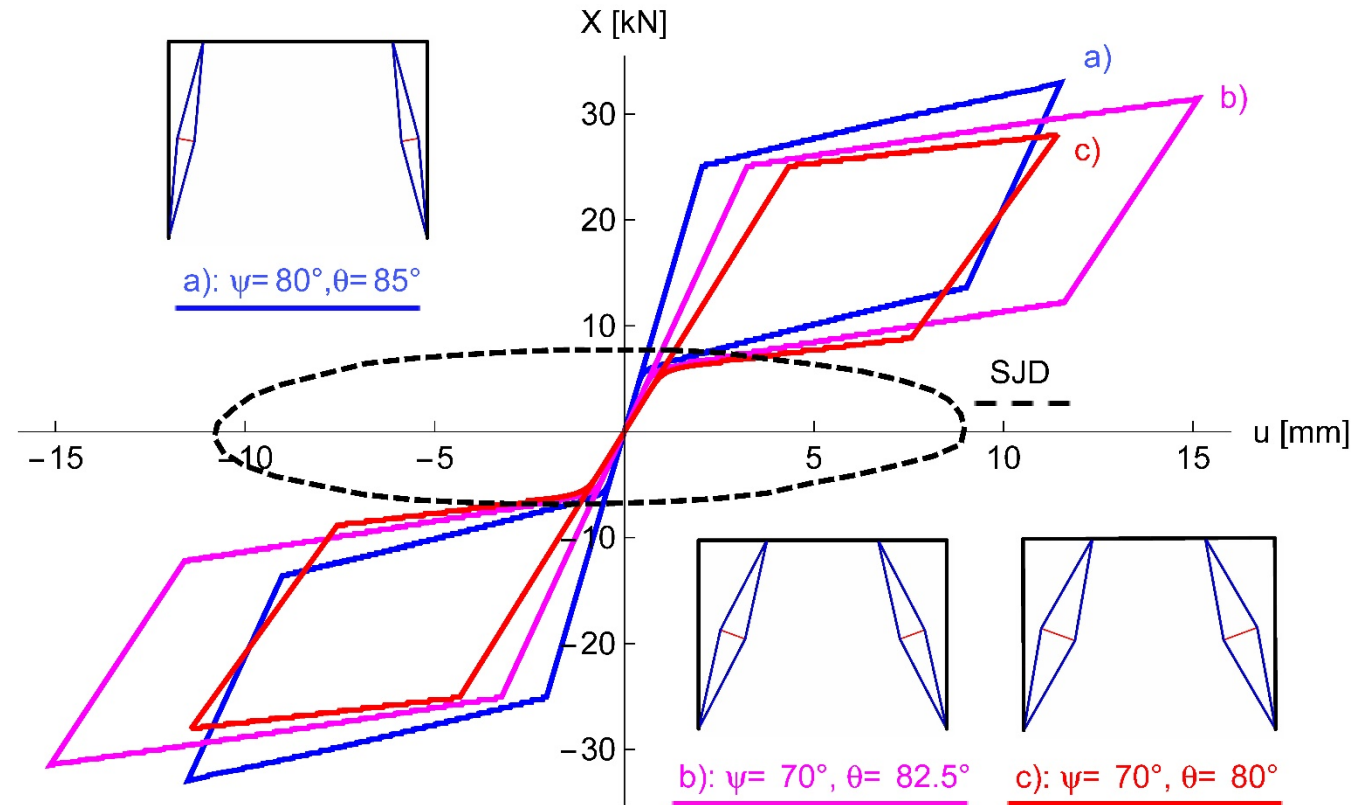
SMAD brace inserted into a timber frame

Lateral displacement amplification factor



Lateral displacement amplification ratio of the SMAD brace for $\delta = 1\%$ and different values of the aspect angles ψ and θ .

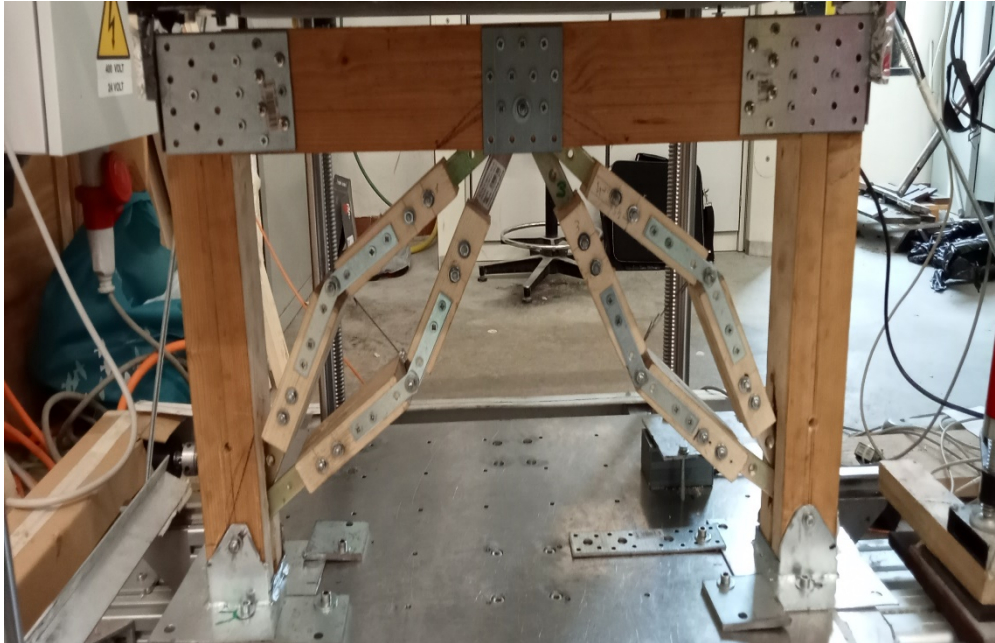
Comparison of Force-displacement response



Force-displacement responses of several bracing systems reinforcing a model frame (SJD: Scissor-Jack Device analyzed in Şığaher, A.N., Constantinou, M.C., Scissor-jack-damper energy dissipation system, Earth-quake Spectra, 19(1) (2003) 133-158)

Use of SMAD braces in timber frames

Insertion of SMAD braces into a reduced scale timber frame



Experimental Setup under Shake table

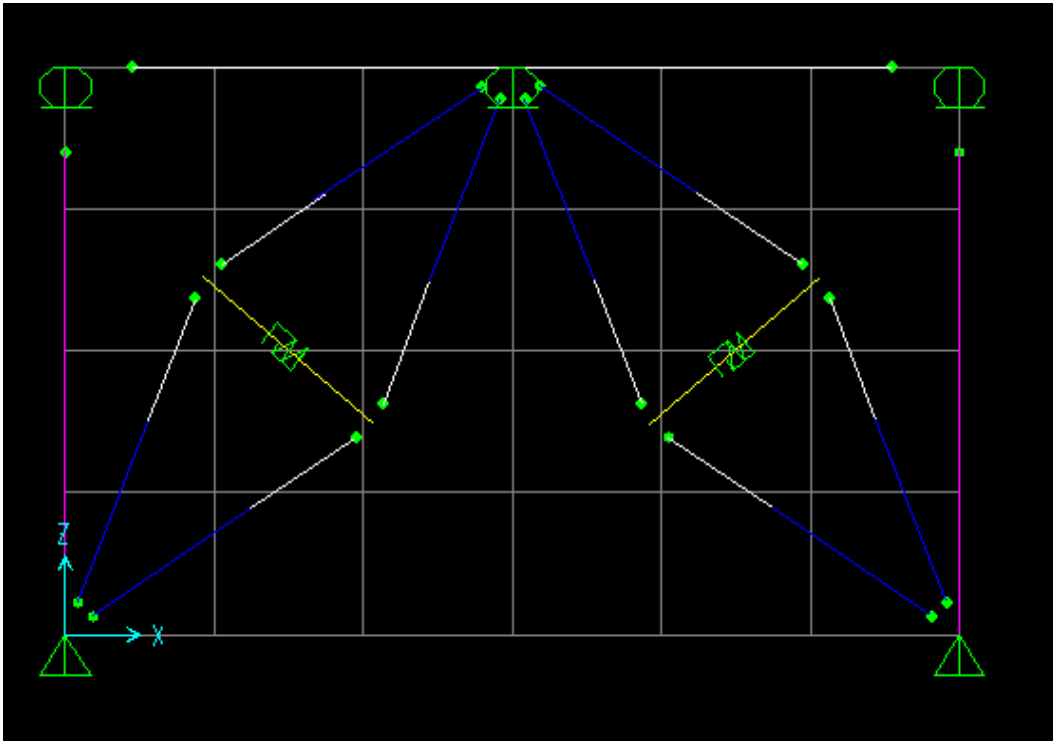
MATERIAL PROPERTIES	ABBREVIATIONS	VALUES
Modulus of Elasticity (MPa)		
Parallel to grain	$E_{0,g,mean}$	11600
Perpendicular to grain	$E_{90,g,mean}$	390
Shear Modulus (MPa)	$G_{g,mean}$	720
Strength Values (MPa)		
Flexural Strength	$f_{m,g,k}$	24
Tensile Strength parallel to grain	$f_{t,0,g,k}$	16.5
Tensile Strength perpendicular to grain	$f_{t,90,g,k}$	0.40
Shear Strength	$f_{v,g,k}$	2.7
Density (kg/m^3)	$\rho_{g,k}$	380

Timber Properties

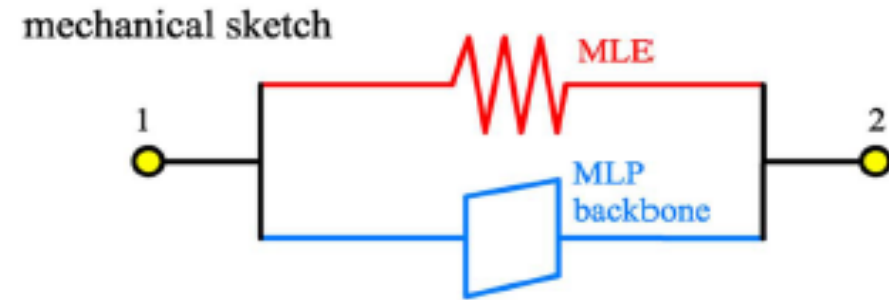
Capacity based design approach is being considered in which all the timber elements are designed as elastic members and the only ductile member is the SMA wire.

Reference: Fraternali, F., de Castro Motta, J. *Mechanics of superelastic tensegrity braces for timber frames equipped with buckling-restrained devices*. International Journal of Solids and Structures, 281, 112414, 2023.

Finite element modeling



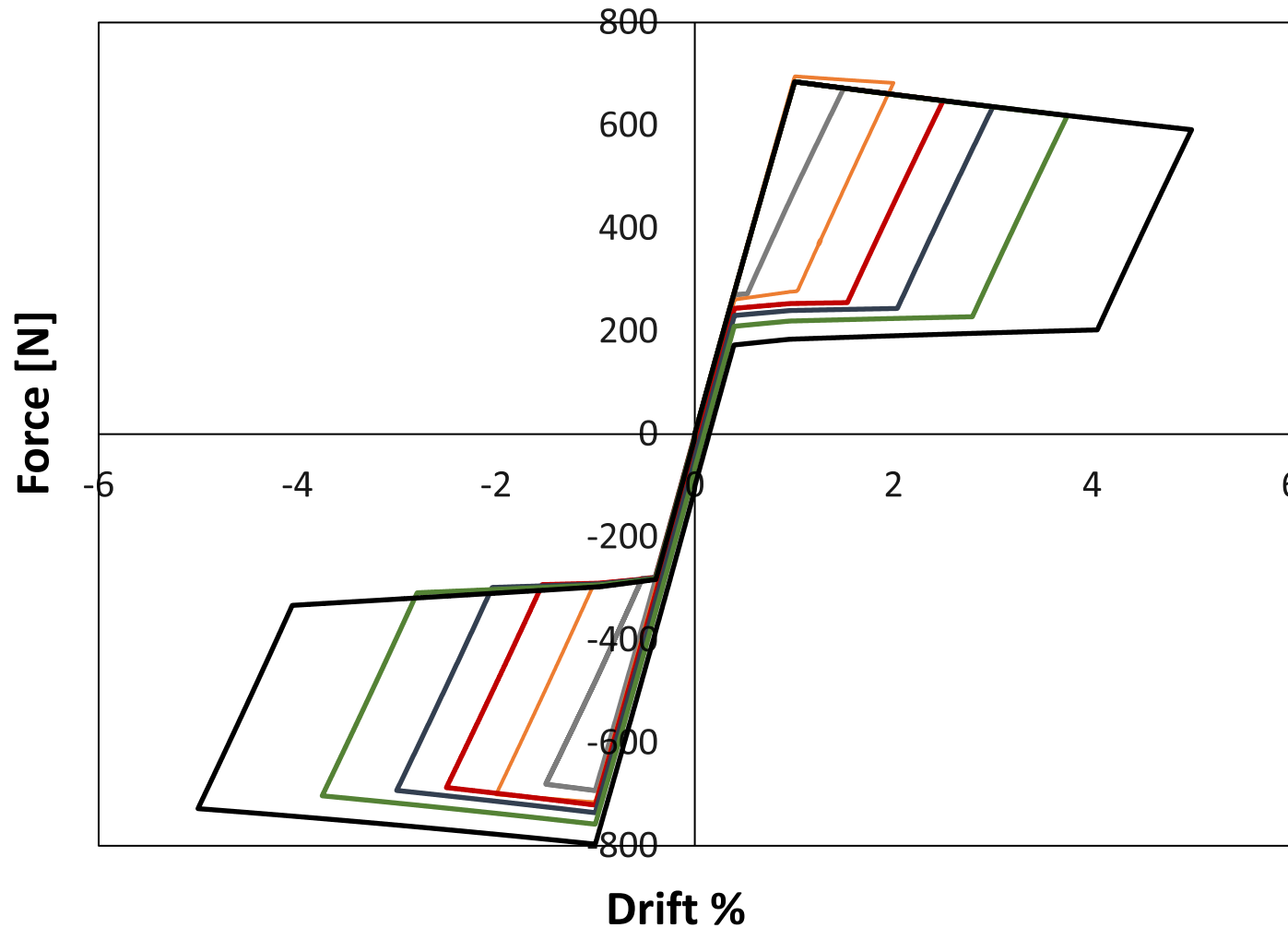
SAP 2000 MODEL



Rheological Model to obtain flag shaped behaviour using Pivot Hysteretical model as suggested by CSi-Wiki modelling

The Multilinear Elastic (MLE) and Multi-Linear Plastic (MLP) are two non-linear links connected in parallel to obtain the desired flag-shaped behaviour

FEM simulations of shake table tests



Superelastic response of the frame illustrating high energy dissipation with negligible residual strains.

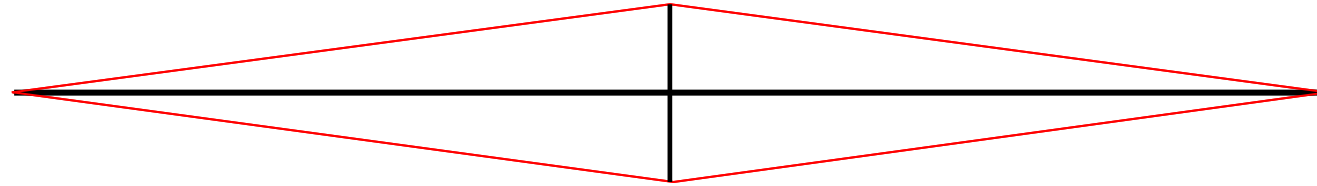
Preliminary experimental results



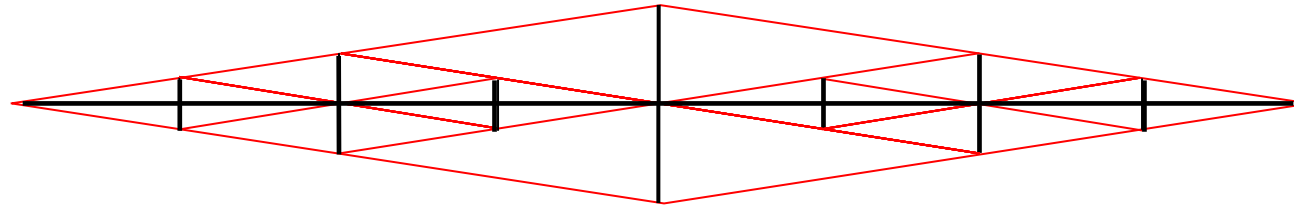
Initial results of shake table tests corresponding to FEM simulations. The tests are currently in progress in order to remove friction effects at the joints.

A scientific paper on this topic is under preparation.

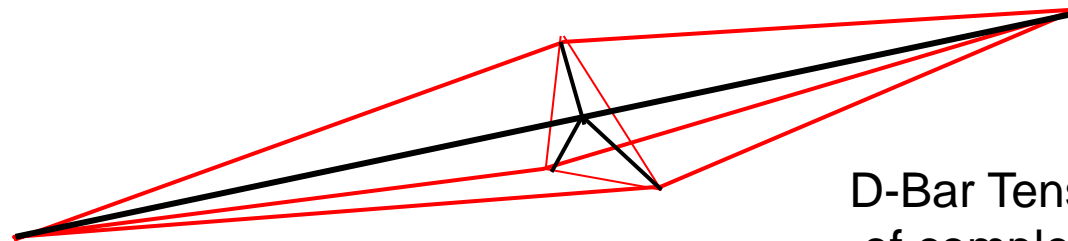
Form-finding procedure based on fractal geometry



D-Bar Tensegrity Brace of complexity $n=1$
(**back** lines are strings, **red** lines are compressive members)

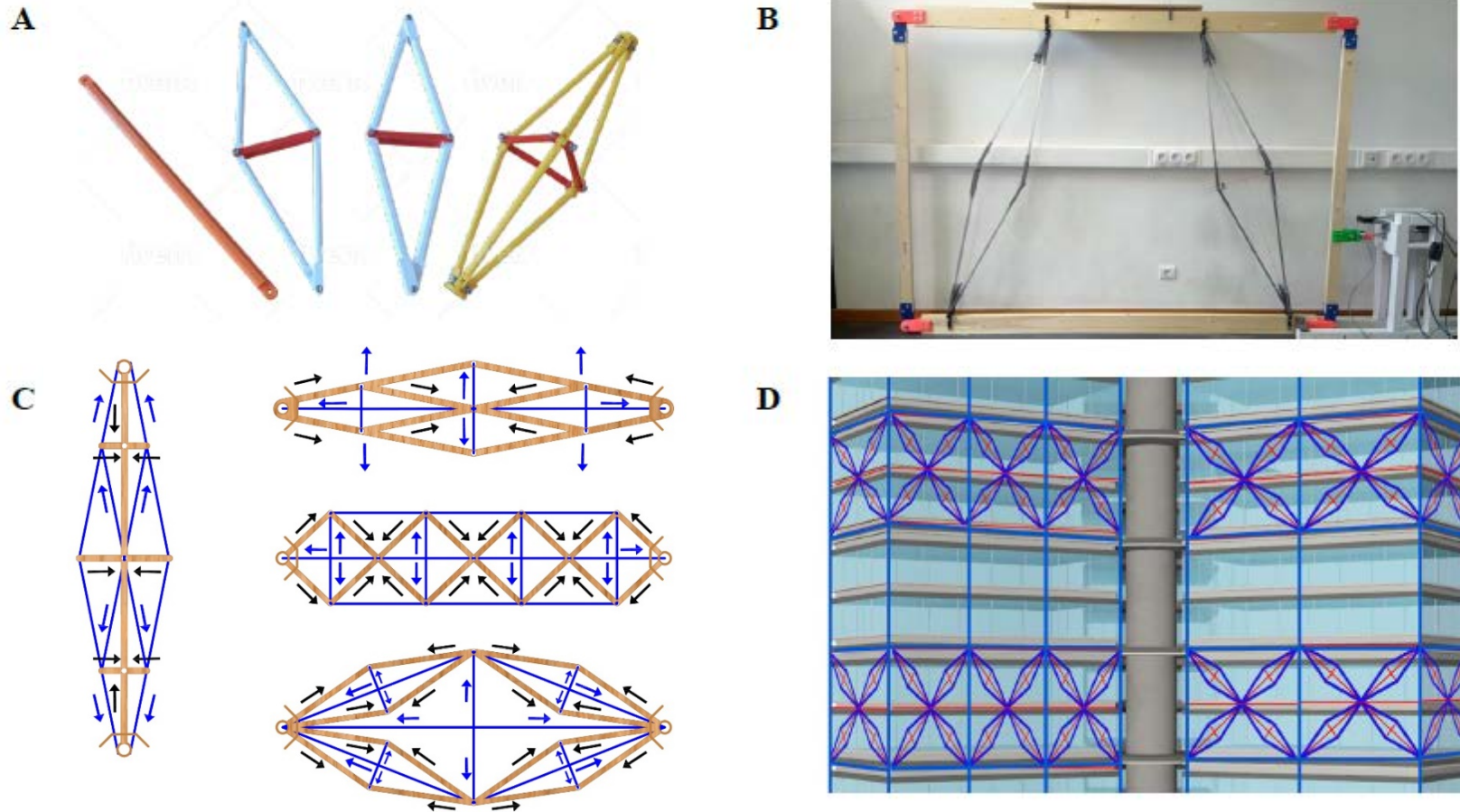


D-Bar Tensegrity Brace of complexity $n=3$
(**back** lines are strings, **red** lines are compressive members)



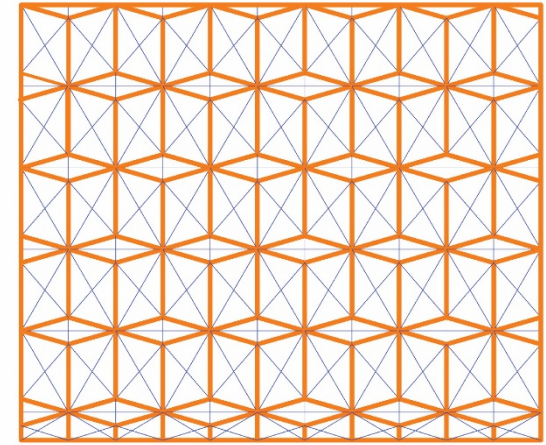
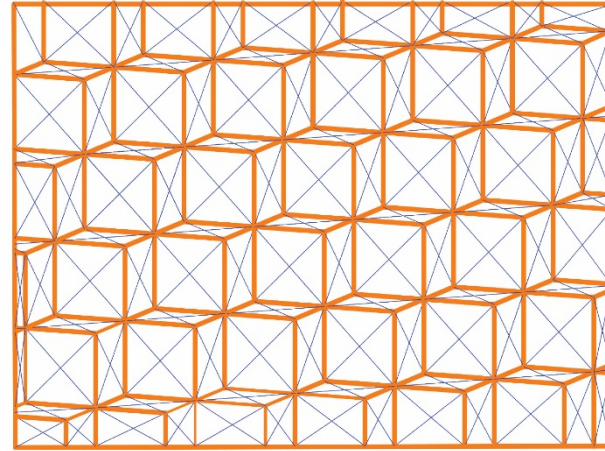
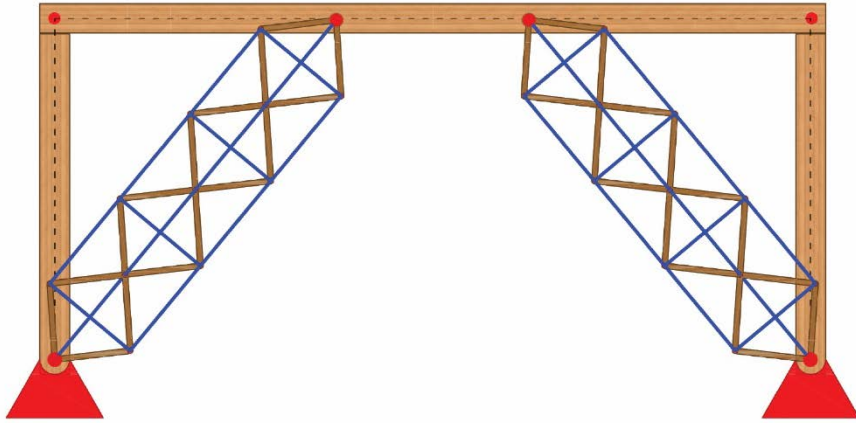
D-Bar Tensegrity Brace
of complexity $n=1$, 3D case

SMAD metamaterials 1/2

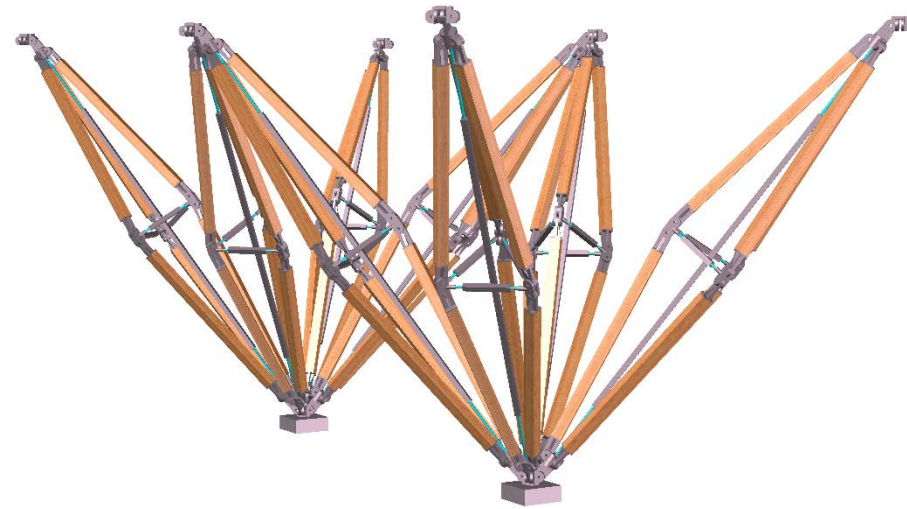


A 3D printed T-brace modules. **B** Timber frame equipped with D-bar-braces under testing. **C** Internally prestressed meta-braces designed through parametric approaches and fractal geometry. **D** Belt trusses with tensegrity architecture

SMAD metamaterials 2/2



Use of metamaterials obtained by tessellating SMAD units to form earthquake-resistant timber frames, walls, and 3D structures



Concluding remarks

The analyzed SMAD bracing systems exhibit high displacement amplification properties, lightweight design to mitigate against buckling, high energy dissipation capacity, and strong re-centering ability.

The main advantages of SMAD braces over viscous dampers derive from the use of SMA cables in place of more widespread dissipation devices, the increase of the buckling resistance with the tapering of the structure, their re-centering capacity, and their displacement amplification properties being based on the geometry of the system rather than the chemical nature of the material.

The use of timber braces merits special attention, since its use engenders significant environmental benefits. The latter mainly derive from the possibility to realize lightweight structures that contribute to reducing the carbon footprint of the building; can be assembled using convenient prefabrication techniques, and are suitable for reuse.

Future research line regard the development of seismic metamaterials tessellating SMAD units over one or multiple directions in space, with recourse to self-similar design approaches.



Thank you for your kind attention

Any queries can be redirected to Baidehi DAS <bdas@unisa.it>